Jeans instability of molecular cloud under the influence of electron inertia and fine Dust particles

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Abstract – The problem of Jeans instability of gaseous molecular cloud in the presence fine dust particles electron inertia and the influence of thermal conductivity. The mathematical form of the problem is performed and a general dispersion relation is obtained using the normal mode analysis with the help of relevant linearized perturbation equations. Furthermore, the wave propagation along parallel and perpendicular to the direction of existing magnetic field has been discussed.

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1. INTRODUCTION

IN modern astrophysical and star formation problems these has been a great deal of interest in investigating various collective process in gaseous plasma, which are ubiquitous in space including diffuse and dense interstellar media, circumstellar shells, ionosphere, dark interiors molecular clouds accretion disks, nova ejecta, star envelopes and the outflow of red giant star. Among various astrophysical objects, molecular clouds, lying in the central region of our galaxy, have drawn the recently great interest of the astrophysicists because they are thought to be the regions of modern star formation. The starting point for modern star cosmogony is that stars are formed and reach a state similar to that of the sun owing to the gravitational condensation of rare field clouds of the gas. The gravitational instability problem of an infinite homogeneous medium was first discovered by Jeans [1]. A detailed contribution of the several gravitational instabilities with different assumptions on the magnetic field and rotation has been given by Chandrasekhar [2]. The importance of thermal conductivity as it is associated with most of the astrophysical situations is an established fact. The problem of thermal instability, arising owing to various heat mechanism of interstellar matter, plays an important role in astrophysical condensations and the formation of prominences through condensation of coronal material. Kato and Kumar [3] have studied the problem of the gaseous plasma incorporating finite thermal conductivity, and in their result, they found that adiabatic speed of sound is being replaced by the isothermal one, much similar to what happens in the absence of magnetic field. Coroniti [4] investigated the dissipative effects of viscosity finite electrical and thermal conductivities on shock waves. Sharma [5, 6, 7, 8] Sharma *et al.* [7] have investigated the effect of fine dust particles (suspended particles) on the onset of Benard convection in hydromagnetics incorporating various parameters. The importance of suspended particles in the study of gravitational instability of magnetized and rotating plasma has been studied by Chhajlani and Sanghavi [9]. Chhajlani and Vyas [10] have investigated the effect of thermal conductivity and suspended particle on the gravitational instability of magnetized rotating plasma through a porous medium. In this connection, many investigators have discussed the gravitational instability of a homogeneous plasma considering the effects of various parameters [Bora and Talwar [11], Chhajlani and Parihar [12], Uberoi [13], Shaikh and Khan [14], Prajapati et al. [15], Pensia et al. [16], Sharma and Chhajlani [17], Patidar et al. [18], Joshi and Pensia [19]]. Sutar and Pensia [20], have discussed the Electron Inertia effects on the Gravitational instability under the influence of FLR Corrections and Suspended Particles.

In the recent ISM observations, it has been established as a fact that comets consist of a dusty 'Snowball' of a mixture of frozen gasses which in the process of their journey changes from solid to gas and vice versa. Recently, Pensia et al. [16], have investigated the role of Coriolis force and suspended particles in the fragmentation of matter in the central region of the galaxy and suggested that Coriolis force and suspended particles play an important role

in the central region of the galaxy. Electron-Inertia Effects on the Transverse Gravitational Instability is analyzed by Uberoi. [14].

Thus, the aim of the present paper is to study the effect of thermal conductivity on the self-gravitational instability of an infinite homogeneous magnetized plasma in the presence of fine dust particles and electron inertia. It is clear, from all the above studies, that none of the authors have carried the Joint study of the effects of fine dust particles, thermal conductivity and electron inertia on the problem of self-gravitational instability of gaseous plasma. Thus, in the present work, we are motivated to investigate the effect of thermal conductivity on the self-gravitational instability of gaseous plasma in the presence of electron inertia and fine dust particles. The result of the present study will help to understand the Interstellar medium structure.

2. LINEARIZED PERTURBATION EQUATIONS

We consider an infinite homogeneous, viscous, Self-gravitating, ionized plasma composed of gas and the fine dust particles (suspended particles) incorporating thermal conducting and finite electron inertia, flowing through a porous medium.

Linearized Perturbation Equations of the Problem are,

$$\frac{\delta \vec{v}}{\delta t} = -\frac{\vec{\nabla}\delta P}{\rho} + \vec{\nabla}\delta \phi + \frac{KN}{\rho}(\vec{u}-\vec{v}) + \vartheta \nabla^2 \vec{v} - \frac{\vartheta}{k_1}\vec{v} + \frac{1}{4\pi\rho}(\vec{\nabla}\times\vec{b})\times\vec{B} \qquad (1)$$

$$\mathcal{E}\frac{\partial \delta\rho}{\partial t} = -\rho\vec{\nabla}.\vec{v} \qquad (2)$$

$$\delta P = C^2 \delta P \qquad (3)$$

$$\nabla^2 \delta\phi = -4\pi G\delta\rho \qquad (4)$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\vec{u} = \vec{v} \qquad (5)$$

$$\lambda\nabla^2\delta T = \rho C_p \frac{\partial\delta t}{\partial t} - \frac{\partial\delta P}{\partial t} \qquad (6)$$

$$\frac{\delta P}{P} = \frac{\delta T}{T} + \frac{\delta\rho}{\rho} \qquad (7)$$

$$\frac{\partial\vec{b}}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{b}) + \frac{C^2}{\omega_{pe}^2}\frac{\partial}{\partial t}\nabla^2\vec{b} \qquad (8)$$

Where, $\vec{v}(v_x, v_y, v_z), \vec{u}(u_x, u_y, u_z)$, $N, \rho, P, \phi, \vec{B}(0,0,B), T, G, \vartheta, k_1, \mathcal{E}, C_p, \lambda, R, m, \rho_s,\ \omega_{pe}, k_s(6\pi\rho r)\ and\ \vec{b}(b_x, b_y, b_z)$ denote respectively, the gas velocity, the particle velocity, the number density of the particle, density of the gas, pressure of the gas, Gravitational potential, magnetic field, temperature, Gravitational constant, kinematic viscosity, permeability, porosity, specific heat at constant pressure, thermal conductivity, gas constant, mass per unit volume of the particles its density, plasma frequency of electron, the constant is the stokes drag formula and the perturbation in magnetic field.

3. DISPERSION RELATIONS

We analyze these perturbations with normal oscillation technique. We find the solution of equation (1)-(8). In a uniform system, we can find a plane-wave solution with all variables varying as,

$$exp\{i(k_x + k_z + \omega t)\} \qquad (9)$$

Where k_x, k_z are the wave numbers of perturbation along the x and z-axis so that $k_x^2 + k_z^2 = k^2$ and the frequency of harmonic disturbances, Using (2)-(9) in (1), we obtain the following algebraic equations for the components

$$M_1 v_x \; + \frac{ik_x}{k^2}\Omega_T^2 s \; = 0 \tag{10}$$

$$M_2 v_y \;\; = 0 \tag{11}$$

$$d_1 v_z \; + \frac{ik_z}{k^2}\Omega_T^2 s \; = 0 \tag{12}$$

The divergence of (1) with the aid of (2)-(9) gives

$$\frac{ik_x v^2 k^2}{A_1} v_x - M_3 s \; = 0 \tag{13}$$

Where $s = \frac{\delta\rho}{\rho}$ the condensation of the medium

$\gamma \; = \; \frac{C_p}{C_v} = \frac{C^2}{C'^2}$ ratio of the specific heat, $V = \frac{B}{\sqrt{4\pi\rho}}$ is the Alfven velocity,

$a \; = \; \frac{kN}{\rho}$ has the dimension of frequency, $\quad \tau \; = \; \frac{m}{k_s}$ is the relaxation time,

$\beta \; = \; \tau a = \frac{\rho_s}{\rho}$ is the mass conservation, $\quad \sigma \; = i\omega$ is the growth rate ofperturbation,

$\Omega_\vartheta \; = \; \vartheta\left(k^2 + \frac{1}{k_1}\right), \quad A_1 \; = \; \sigma f, \quad f \; = \left(1 + \frac{C^2 k^2}{\omega_{pe}^2}\right), \theta_k = \frac{\lambda}{\rho C_p}$ is the thermometric Conductivity,

C and C′ are the adiabatic and isothermal velocities of sound.

$d_1 = \left(\sigma + \Omega_\vartheta + \frac{\beta\sigma}{\sigma\tau+1}\right), \Omega_j^2 = (C^2 k^2 - 4\pi G\rho), \Omega_{j'}^2 = (C'^2 k^2 - 4\pi G\rho), \; \Omega_T^2 = \frac{\sigma\Omega_j^2 + \theta_k \Omega_{j'}^2}{\sigma+\theta_k},$

$M_1 = \left(d_1 + \frac{V^2 k^2}{A_1}\right), M_2 = \left(d_1 + \frac{V^2 k_z^2}{A_1}\right), M_3 = \left(\mathcal{E}\sigma d_1 + \Omega_T^2\right),$

The nontrivial solution of the determinant of the matrix obtained from (11)-(13) with $(v_x, \; v_y, \; v_z,)$ having various coefficients, that should vanish is to give the following dispersion relation.

$$d_1\left[\sigma d_1 \{M_1 M_2\} + \frac{\Omega_T^2}{\mathcal{E}}\{M_2^2\}\right] = 0 \tag{15}$$

The dispersion relation (15) shows the combined influence of fine dust particles thermal conductivity, finite electron inertia, magnetic field, viscosity porosity of the self-gravitational instability of a homogeneous plasma. If we ignore the effect of finite electron inertia then (15) reduces to Chhajlani and Vyas [10]. The present results are also similar to those of Chhajlani and Sanghvi [9].

4. ANALYSIS OF THE DISPERSION RELATION

4.1 Longitudinal mode of propagation ($K \parallel B$)

For this case, we assume that all the perturbations and longitudinal to the direction of the magnetic field($i.e. k_z = k, \; k_x = 0$). Thus the dispersion relation (15) reduces in the simple form to give

$$d_1\left(\sigma d_1 + \frac{\Omega_T^2}{\mathcal{E}}\right)\left(d_1 + \frac{k^2 V^2}{A_1}\right)^2 = 0 \tag{16}$$

This dispersion relation is the product of three independent factors. These factors show the mode of propagations incorporating different parameters as discussed below. The first factor equated to zero gives,

$$\tau\sigma^2 + \sigma\{1 + \tau(a + \theta_k)\} + \theta_k = 0 \tag{17}$$

The first factor of this dispersion relation represents stable mode. The second factor equated to zero gives,

$$\sigma^4\tau + \sigma^3\{1 + \tau(a + \Omega_\vartheta + \theta_k)\} + \sigma^2\left[(\Omega_\vartheta + \theta_k) + \tau\left\{\frac{\Omega_j^2}{\varepsilon} + \theta_k(a + \Omega_\vartheta)\right\}\right] + \sigma\left(\frac{\Omega_j^2}{\varepsilon} + \Omega_\vartheta\theta_k + \tau\theta_k\frac{\Omega_{j'}^2}{\varepsilon}\right) + \theta_k\frac{\Omega_{j'}^2}{\varepsilon} = 0 \quad (18)$$

This dispersion relation shows the combined influence of fine dust particles, viscosity, porosity and thermal conductivity on the self-gravitational instability of the hydromagnetic fluid plasma in the absence of magnetic field and finite electron inertia. The Third factor equated to zero gives,

$$\sigma^8\tau^2 f^4 + A_7\sigma^7 + A_6\sigma^6 + A_5\sigma^5 + A_4\sigma^4 + A_3\sigma^3 + A_2\sigma^2 + A_1\sigma + A_0 = 0 \qquad (19)$$

This is Eight-degree polynomial equations and shows the combined influence of fine dust particles, viscosity, porosity, permeability, finite electron inertia, magnetic field and thermal conductivity in the longitudinal mode of propagation. Where the coefficients are very lengthy and are emitted, the constant term is given as

$$A_0 = K^4 V^4$$

4.2 TRANSVERSE MODE OF PROPAGATION ($K \perp B$)

For this case, we assume all the perturbations transverse to the direction of the magnetic field ($i.e. k_x = k, \quad k_z = 0$).

Thus the dispersion relation (15) becomes

$$d_1^2\left[\sigma\, d_1^2 + d_1\left\{\frac{V^2 k^2\sigma}{A_1} + \frac{\Omega_T^2}{\varepsilon}\right\}\right] = 0 \qquad (20)$$

The dispersion relation (20) shows the influence of finite electron inertia, the presence of finite dust particles, viscosity, magnetic field and thermal conductivity on the self-gravitational instability of infinite homogeneous plasma. The first factor of this dispersion relation is stable mode as discussed in the previous case and the second factor of the dispersion relations (20) simplification written as,

$$\tau^2\sigma^6 f + \sigma^5\tau f[2\{1 + \tau(\beta + \Omega_\vartheta)\} + \tau\theta_k] + \sigma^4\left[\tau^2\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2\right) + 2\tau\Omega_\vartheta f + \{1 + \tau(\beta + \Omega_\vartheta)\}\{1 + \tau(\beta + \Omega_\vartheta + 2\theta_k)\}f\right]$$

$$+ \sigma^3\left[\tau^2\theta_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right) + \tau\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + 2\Omega_\vartheta\theta_k f\right)\right.$$

$$\left. + \{1 + \tau(\beta + \Omega_\vartheta)\}\left\{(2\Omega_\vartheta + \theta_k)f + \tau\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + \Omega_\vartheta\theta_k f + \beta\theta_k f\right)\right\}\right]$$

$$+ \sigma^2\left[f\Omega_k^2 + \{1 + \tau(\beta + \Omega_\vartheta)\}\left\{\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + 2\Omega_\vartheta\theta_k f + \tau\theta_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right)\right\}\right.$$

$$\left. + \tau\left\{\Omega_\vartheta\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2\right) + \theta_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right)\right\}\right]$$

$$+ \sigma\left[\Omega_\vartheta\left(\frac{\Omega_j^2}{\varepsilon}f + k^2V^2 + \Omega_\vartheta\theta_k f\right) + \tau\theta_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right) + \theta_k\{1 + \tau(\beta + \Omega_\vartheta)\}\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right)\right]$$

$$+ \Omega_\vartheta\theta_k\left(\frac{\Omega_{j'}^2}{\varepsilon}f + k^2V^2\right) = 0 \qquad (23)$$

This is Six-degree polynomial equations and shows the combined influence of fine dust particles, porosity, permeability, viscosity, electron inertia, magnetic field and thermal conductivity in the transverse mode of propagation. This is the Alfven mode which is modified by the presence of finite electron inertia, fine dust particles, viscosity, permeability, porosity and thermal conductivity.

5. CONCLUSION

In this paper, we have investigated the Jeans instability of molecular cloud under the influence of finite electron inertia and presence of fine dust particles and the magnetic field incorporating the effect of thermal conductivity, viscosity, permeability and porosity of the medium. The general dispersion relation is obtained, which is modified due to the presence of these parameters. This dispersion relation is reduced for the longitudinal and transverse mode of propagation. Owing to the inclusion of the thermal conductivity the isothermal sound velocity is replaced

by the adiabatic velocity of sound. The effect of the viscosity parameter is found to stabilizing the system in both the longitudinal and the transverse mode of propagation. In the transverse mode of propagation, we obtained Alfven mode which is modified by the presence of finite electron inertia, fine dust particles, porosity, permeability, viscosity and thermal conductivity.

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